plane parallel to the electric vector and let both ends of the waveguide be terminated by a perfect short circuit. Let it be assumed that the cavity thus formed has perfectly reflecting walls at all frequencies in the spectrum under consideration. In this case, the material medium consists of the particles in the gas discharge. The electrons owing to the applied external field and their small mass have by far higher velocities than the other particles in the gas discharge. Since, on the other hand, the interaction energy between radiation and a charged particle is proportional to the velocity of the particle, the contribution of the electrons to the radiation energy will be much more important than that of any other particles in the gas discharge. If, in addition, the electron distribution is Maxwellian—and only Maxwellian, neglecting quantum effects—the concept of electron temperature T_e is meaningful and the radiation energy density in the cavity will follow (1) with

$$T = T_e. \tag{2}$$

It can be shown that the noise power available from a black body is given by the quantum theory form of Nyquist's theorem

$$dP = h\nu[\exp(h\nu/\kappa T) - 1]^{-1} \cdot d\nu. \tag{3}$$

In our case the power available using (2) is

$$dP = h\nu[\exp(h\nu/\kappa T_e) - 1]^{-1} \cdot d\nu. \tag{4}$$

So far it has been assumed that the walls were reflecting perfectly at all frequencies. In fact, perfect reflection obtains in the microwave region and therefore (4) holds in that frequency range.

For microwave frequencies one has

$$h\nu/\kappa T_e \ll 1 \tag{5}$$

so that (4) reduces to

$$dP = \kappa \cdot T_e \cdot d\nu. \tag{6}$$

The first part of Mumford's hypothesis is then valid.

It is seen from (4) that the frequency corresponding to maximum power radiation is a function of the electron temperature T_e. On the other hand (4) holds irrespective of the particular properties associated with a given material in the cavity. This is a direct result of the second law of thermodynamics. It therefore follows that part two of Mumford's hypothesis is not valid.

Acknowledgment

Thanks are extended to Professor J. C. Slater, Professor Ladislas Goldstein, and Dr. E. Schloemann for stimulating discussions.

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Design of Aperture-Coupled Filters*

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Summary—A procedure for the design of aperture-coupled filters is presented, based on the theory of conventional coupled circuits. This design procedure accounts for the relatively low insertion loss of aperture-coupled filters as compared with other known designs of microwave filters. The factors which contribute to this low value of insertion loss are the following:

1) Use of a high Q-mode configuration such as a cylindrical cavity in the TE_{011} mode.

2) Aperture coupling of elements eliminating the losses of impedance transforming sections of transmission line.

3) A mechanical design which eliminates joints at critical points and also provides control over interior surface finishes.

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I. Introduction

Many system applications require band-pass filters at microwave frequencies having a low insertion loss. While band-pass filters can be built by cascading resonant cavities formed by irises in rectangular waveguide such as described by Mumford,[1] the insertion loss of this filter structure becomes prohibitively large for many applications at X band and higher frequencies.

[1] W. W. Mumford, "Maximally flat filters in waveguide," *Bell Sys. Tech. J.*, vol. 27, pp. 684–713; October, 1948.

To achieve a lower insertion loss, a filter structure was developed which involves the coupling of cylindrical cavities operating in the TE_{011} mode through coupling apertures in the thin wall separating two adjacent cavities. By holding the thickness of this wall to 0.025 inch, it was possible to reduce materially the insertion loss through this filter structure, which will be called an aperture-coupled filter. For example, a four-section aperture-coupled filter was built at K band which had an insertion loss of 0.3 db, while the comparable iris-in-waveguide structure had an insertion loss of approximately 2.5 db.

The purpose of this paper is to describe the procedure for the design of aperture-coupled filters and to give experimental results on their performance.

II. Coupling to a TE_{011}-Mode Cylindrical Cavity

The basic element used in the design of aperture-coupled filters is a resonant cavity. A photograph of the cylindrical cavity used in the TE_{011}-mode configuration is shown in Fig. 1 with requisite coupling holes. The wall thickness at the coupling holes is 0.020 inch, while the accompanying plunger allows the cavity to be tuned ± 10 per cent of center frequency.

The TE_{011} mode was selected because of its very high unloaded Q, but any other normal mode can be chosen. The resonant wavelength of a TE_{011}-mode cavity is given by the expression[2]

$$\lambda = \frac{2}{\sqrt{\left[\frac{7.664}{\pi D}\right]^2 + \left[\frac{1}{L}\right]^2}} \tag{1}$$

where D is the diameter of the cavity and L, its length. However, for optimum Q, it is desirable to have D equal to L, so that if the resonant wavelength is known, the diameter of the cavity is given by the following:

$$D = 1.318\lambda. \tag{2}$$

Since the cavity dimensions are known, only the dimensions of the apertures which couple the cavities to the waveguide, or which couple adjacent cavities together, need to be determined. It should be clear that it is the dimensioning of these coupling apertures which finally determines the band-pass filter characteristic.

An analysis of the loaded Q, Q_L, of a cylindrical cavity oscillating in the TE_{011} mode, coupled to a rectangular waveguide, is given in Appendix I. The basic relation finally stated in (21) for an infinitesimally thin circular coupling aperture centrally located on the curved wall of the cavity is

$$\frac{1}{Q_L} = \frac{1.32\lambda^2 d^6}{ab\lambda_g D^4 L} \tag{3}$$

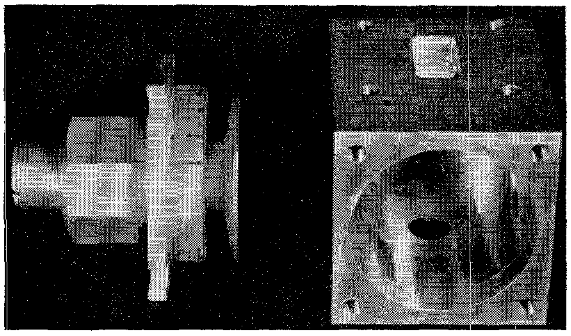

Fig. 1—Prototype TE_{011}-mode filter section.

where d is the diameter of the coupling hole, a and b the width and height of the waveguide, and λ_g the guide wavelength in the rectangular waveguide. This equation is the result of applying Bethe's theory of diffraction by small holes.[3] Consequently, the result is valid only for the case in which $d \ll \lambda$. This result also neglects heat losses in the cavity walls, so that it will yield a value of Q_L slightly higher than observed.

The coefficient of coupling γ between two adjacent TE_{011}-mode cavities coupled together by a common hole of diameter d may be expressed approximately by

$$\gamma = \frac{d_1^3}{2\pi D^2 L}. \tag{4}$$

This approximate relation is given by Bethe,[3] and allows filter theory as developed for circuits which are usually expressed in terms of parameters Q_L and γ (*i.e.*, the loaded Q of the resonant circuit and the coupling coefficient between adjacent resonant circuits) to be applied to the design of a microwave filter in a quantitative manner.

III. Specific Cases and Experimental Data

A. A Single Resonant Cavity

Eq. (3) can be applied directly to the design of a band-pass filter. If the filter is to consist of a single cavity with two equal but diametrically opposite coupling holes, the loaded Q of the structure will then be halved so that

$$\frac{1}{Q_l} = \frac{2.64\lambda^2 d^6}{ab\lambda_g D^4 L}. \tag{5}$$

Measurements were made on the structure shown in Fig. 1 for various values of coupling hole diameter. The experimental points and the theoretical curve, as obtained from (5), are given in Fig. 2 for the case in which

$\lambda = 0.875$ inch
$a = 0.622$ inch
$b = 0.311$ inch
$D = 1.250$ inches
$L = 0.860$ inch.

[2] T. Moreno, "Microwave Transmission Design Data," McGraw-Hill Book Co., Inc., New York, N. Y., p. 219; 1948.

[3] H. A. Bethe, "Theory of diffraction by small holes," *Phys. Rev.*, vol. 66, pp. 163–181; October, 1944.

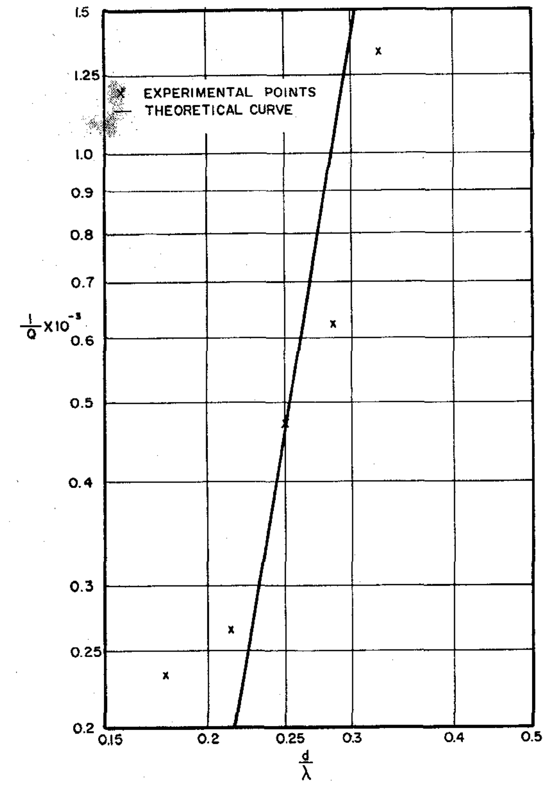

Fig. 2—$1/Q_L$ vs d/λ for a cylindrical TE_{011}-mode cavity coupled to a rectangular waveguide through two diametrically opposite and equal coupling holes.

For small values of d/λ, the experimental points give higher values of $1/Q$ than predicted by the curve of (5). This is to be expected, since the losses in the cavity walls have not been considered and will be the limiting factors in determining $1/Q$ for small values of d/λ. For the larger values of d/λ the agreement is better and indicates that Bethe's small-hole theory is a satisfactory approximation for holes of the order of $\lambda/4$ in diameter.

B. Two Resonant Cavities Coupled Together

The structure that is visualized, is the coupling of two TE_{011}-mode cavities, as shown in Fig. 3, by a common-coupling hole.

This circuit can be treated in terms of the equivalent circuit of a double-tuned coupled circuit treated by Terman.[4] The equivalent circuit is illustrated in Fig. 4.

The loaded Q of the primary and secondary is determined by the usual relations

$$Q_p = \frac{\omega_0 L_p}{R_p} \qquad Q_s = \frac{\omega_0 L_s}{R_s} \cdot \tag{6}$$

For the case of the microwave structure, the loaded Q of the primary and secondary is determined from (3) by substituting the dimension of the particular cavity, waveguide, and coupling hole. The only other variable determining the filter characteristic, is the diameter of the coupling hole which couples two adjacent cavities.

[4] F. E. Terman, "Radio Engineering," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 58–66; 1947.

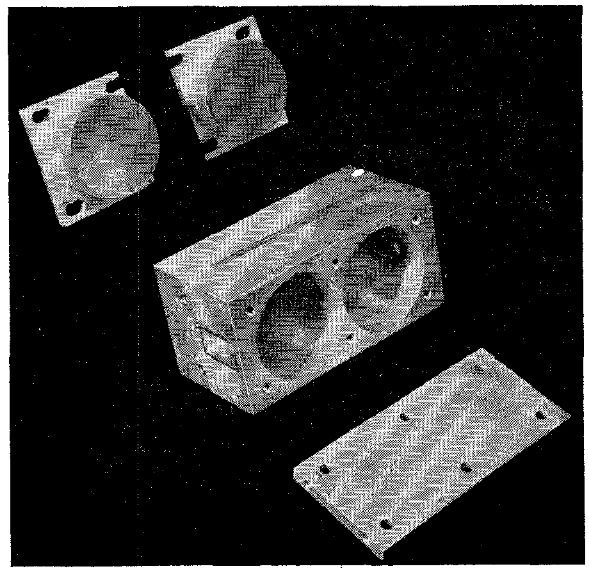

Fig. 3—A two-section aperture-coupled filter.

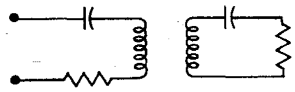

Fig. 4—Equivalent of a double-tuned circuit.

This diameter can be determined from (4) once the value of coupling coefficient γ is known. γ determines whether the filter characteristic is overcoupled, critically coupled, or undercoupled. The particular value of γ can be selected from universal characteristic curves such as those given by Terman.[4]

An example of this design procedure is given for the following case of a two-cavity band-pass filter at X band. A filter having a half-power bandwidth of 3.0 mc centered at 9000 mc was desired. Two TE_{011}-mode cavities, 1.94 inches in diameter and 1.15 inches in length, were used. A curve of the half-power bandwidth vs coupling hole diameter for a single cavity coupled to waveguide, is given in Fig. 5. The cavities were coupled to RG-52/U waveguide at the input and output of the filter and were coupled together by a common coupling hole. From universal resonance curves it was established that the single cavity coupled to a waveguide must have a half-power bandwidth of 2.0 mc. Fig. 5 indicates that a hole diameter of 0.323 inch is required to couple the cavities to the input and output waveguides. The only remaining quantity to be determined was the diameter of the hole coupling the two cavities together. Substituting in (4), the coefficient of coupling for critical coupling, $\gamma = 1/Q_L = 2/9000$, and solving for the hole diameter yielded a value of 0.176 inch. Experimental tests showed that this value was somewhat too small, so that the filter had an undercoupled characteristic. A direct method of determining the coupling coefficient of a given coupling hole is given in Appendix II. This experimental method led to a coupling hole of 0.182 inch in diameter and a filter characteristic corresponding to a critically coupled double-resonant circuit.

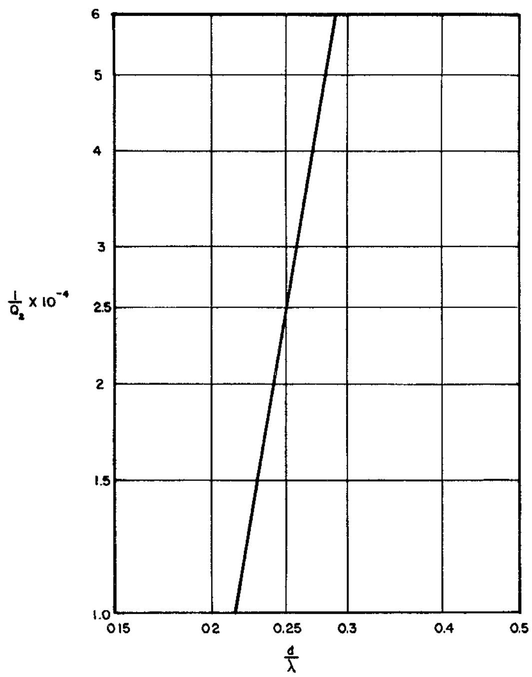

Fig. 5—$1/Q_L$ vs d/λ for a cylindrical TE_{011}-mode cavity coupled to a rectangular waveguide through a single coupling hole.

IV. Insertion Loss in Aperture-Coupled Filters

The insertion loss in any microwave component usually can be attributed to a copper loss in the conducting walls or to an absorption of energy by some additional dissipative element within the device. As no dissipative element is introduced into the resonant cavities of the filters in question, the remaining cause of loss is simply the heat loss on the cavity walls and in the coupling irises. Therefore, the problem is that of deducing a simple expression for this insertion loss.

Let us examine the basic definition of Q as given in (13) of Appendix I.

$$Q = \frac{\omega \text{ (energy stored in the circuit)}}{\text{average power loss}}. \quad (7)$$

For a given cavity, the energy storage should be a function of the mode of oscillation and, therefore, independent of loading. Average power loss, however, can be ascribed to losses within the cavity, and to losses caused by a transfer of power out of a cavity. Consider the ratio Q_L/Q_u, where Q_u is the cavity Q with no power coupled to an outside circuit, and Q_L is as defined in Appendix I.

$$\frac{Q_L}{Q_u} = \frac{\omega W_s}{P_t} \cdot \frac{P_h}{\omega W_s} = \frac{P_h}{P_t} \quad (8)$$

where P_h is the average power lost by rf heating in the walls and P_t is the average power transferred to an external circuit. If Q_L is taken as the quantity defined by (21), then the ratio $Q_L/2Q_u$ represents the ratio of power lost in rf heating to the power transferred through a cavity having two symmetrically placed coupling holes. The power loss though the cavity L in decibels is given by

$$L = 10 \log_{10} \left[\frac{1}{1 - \frac{Q_L}{2Q_u}} \right]. \quad (9)$$

Since $Q_L/2Q_u \ll 1$,

$$L \cong 10 \log_{10} \left[1 + \frac{Q_L}{2Q_u} \right]. \quad (10)$$

This expression represents the power loss in just one cavity. In order to calculate the total power loss it is necessary to sum over the total number of cavities, N.

$$L_T \cong \sum_{i=1}^{N} 10 \log_{10} \left[1 + \frac{Q_{Li}}{2Q_{ui}} \right]. \quad (11)$$

In calculating the above power loss, Q_L is obtained by substituting physical dimensions of the cavity in (21), while values Q_u can be calculated or obtained from universal curves as given by Moreno.[2] Eq. (11) can be applied to the X-band filter described in Section III-B. The unloaded Q for the single cavity is about 20,000, while the loaded Q should be about 2100. Substituting into (11), one obtains

$$L_t = \sum_{i=1}^{2} 10 \log_{10} \left[1 + \frac{2100}{40,000} \right] = 0.44 \text{ db}. \quad (12)$$

The measured insertion loss was 0.5 db, which is in good agreement with the computed value. Eq. (11) is an approximation in which Q_L is considered to be the value of Q obtained when a cavity is loaded by coupling it to an external waveguide circuit. In general, this is not the case because the cavity in question may be coupled to two adjacent cavities. In spite of this approximation, the results obtained using (11) agree quite well with measured values of insertion loss, indicating that the losses in the coupling holes or irises are essentially negligible in comparison with the rf losses over the cavity walls.

Appendix I

Derivation of Loaded Q of a TE_{011}-Mode Cylindrical Cavity Coupled to a Rectangular Waveguide

The basic relation for the Q of any circuit can be expressed as follows:[5]

$$\frac{1}{Q} = \frac{\text{average power loss}}{\omega \text{ (energy stored in the circuit)}} = \frac{S}{\omega W_s}. \quad (13)$$

[5] S. Ramo and J. R. Whinnery, "Fields and Waves in Modern Radio," John Wiley & Sons, Inc., New York, N. Y., p. 9; 1944.

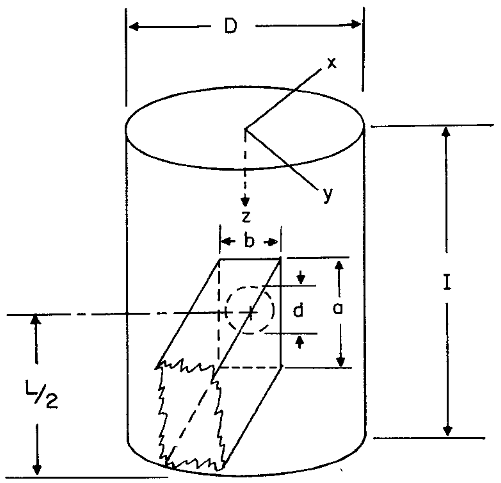

Fig. 6—Cavity and waveguide configuration with cavity system of coordinates.

Assume that a cylindrical cavity of diameter D and length L is coupled to a rectangular waveguide of width a and height b by a hole of diameter d. The configuration is shown in Fig. 6.

Assume that the TE_{011} mode is excited in the cylindrical cavity with the following field components:

$$\begin{aligned} E_\theta &= J_1(k_1 r)\sin\frac{\pi z}{L} \\ E_p &= E_Z = H_\theta = 0 \\ H_p &= \frac{\pi}{j\omega\mu L} J_1(k_1 r)\cos\frac{\pi z}{L} \\ H_Z &= \frac{-k_1}{j\omega\mu} J_0(k_1 r)\sin\frac{\pi z}{L} \end{aligned} \tag{14}$$

where $k_1 = 7.664/D$, and a time variation of exp $(j\omega t)$ is assumed. The energy stored in the cavity W_s is given by

$$W_s = \frac{\epsilon}{2}\int_v E_\theta^2 dV \tag{15}$$

where V is the volume of the cavity. Substituting (14) into (15) the expression for the stored energy can be evaluated as

$$W_s = \frac{\pi\epsilon D^2 L}{16} J_0^2(3.832) \tag{16}$$

where J_0 is the Bessel function of the first kind of order zero.

According to Bethe,[3] the average power lost through a window can be expressed as

$$S = \frac{k^2}{2S_a} \mid PE_{on}E_{an} + jM_1 H_{ol} H_{al} + jM_2 H_{om} H_{am} \mid^2 \tag{17}$$

where P, M_1, and M_2 are the polarizabilities of the window which depend on the geometrical configuration of the window. S_a is a normalizing constant. The fields with subscript 0 are the cavity fields, while the fields with subscript a are waveguide fields. The subscripts n, l, m refer to the principal axes of the window. The geometrical orientation is shown in Fig. 7. The waveguide fields for the dominant TE_{10} mode are given as

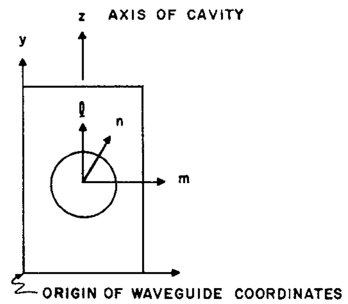

Fig. 7—System of coordinates in the window.

$$\begin{aligned} H_z &= H_1 \cos\frac{\pi x}{a} \\ H_x &= \frac{j2aH_1}{\lambda_g}\sin\frac{\pi x}{a} \\ E_y &= \frac{-jn2aH_1}{\lambda}\sin\frac{\pi x}{a}\cdot \end{aligned} \tag{18}$$

The average power transmitted down the waveguide is given by

$$S_a = \frac{2a^3 b\eta}{2\lambda\lambda_g} H_1^2. \tag{19}$$

For a circular window, the polarizabilities are $M_1 = M_2 = d^3/6\eta$, and $P = d^3/3\eta$.[6]

The expression for the average power lost through the window can now be obtained by substituting the values for the polarizabilities, (18), (19), and (14) into (17) to obtain

$$S = \frac{1.63\lambda d^6}{abD^2\lambda_g\eta} J_0^2(3.832). \tag{20}$$

The final form of the expression for $1/Q$ can now be obtained by substitution (15) and (20) into (13).

$$\frac{1}{Q_L} = \frac{1.32\lambda^2 d^6}{ab\lambda_g D^4 L}\cdot \tag{21}$$

This equation expresses the loaded Q of the cavity in terms of the waveguide and cavity and coupling hole dimensions. A correction can be made to account for heat losses, but it is usually not warranted because of the high unloaded Q of the cavity.

[6] A. T. Starr, "Radio and Radar Technique," Pitman Publ. Corp., New York, N. Y.; 1953. See p. 157 for the polarizabilities in mks units.

Appendix II

Experimental Method of Determining Coupling Coefficient

Consider two identical structures consisting of a waveguide coupled to a cavity, as shown in Fig. 6, to be coupled together through a common coupling hole. The diameter of this coupling hole can be approximated using (4), but an experimental check of the value is highly desirable.

The equivalent circuit under consideration is illustrated in Fig. 8.

The impedance looking into the input waveguide is

$$Z_1 = \frac{(\omega M_1)^2}{Z + \dfrac{(\omega M)^2}{Z}} \tag{22}$$

where

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right).$$

At resonance

$$Z_1 = \frac{(\omega M_1)^2}{R[1 + (\gamma Q_L)^2]} \tag{23}$$

where γ is the coupling coefficient, and $Q_L = \omega_0 L/R$ can be obtained from (21).

The reflection coefficient is given by the following relation:

$$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0} \tag{24}$$

where Z_0 is the characteristic impedance of the input waveguide. Substituting (23) into (24) yields

$$\Gamma = \frac{(\omega M_1)^2 - RZ_0[1 + (\gamma Q_L)^2]}{(\omega M_1)^2 + RZ_0[1 + (\gamma Q_L)^2]}\cdot \tag{25}$$

If the first cavity in the circuit is tuned to resonance and the second is detuned, then $k = 0$ and

$$\Gamma = \frac{(\omega M_1)^2 - RZ_0}{(\omega M_1)^2 + RZ_0}\cdot \tag{26}$$

If critical coupling is attained, $\gamma_c = 1/Q_L$ and it follows that

$$\Gamma_c = \frac{(\omega M_1)^2 - 2RZ_0}{(\omega M_1)^2 + 2RZ_0}\cdot \tag{27}$$

Solving (26) for $(\omega M_1)^2$ yields

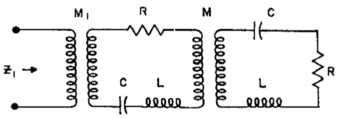

Fig. 8—Equivalent circuit of a two-cavity filter.

$$(\omega M_1)^2 = RZ_0 \frac{[1 + \Gamma_0]}{[1 - \Gamma_0]} = RZ_0 S_0 \tag{28}$$

where S_0 is the input vswr that is measured with the second resonator detuned. Substituting this result into (25) yields

$$\Gamma = \frac{S_0 - [1 + (\gamma Q_L)^2]}{S_0 + [1 + (\gamma Q_L)^2]}\cdot \tag{29}$$

Eq. (29) can now be solved for the γQ_L to give

$$\gamma Q_L = \frac{\gamma}{\gamma_c} = \sqrt{\frac{S_0 - 1 - \Gamma(S_0 + 1)}{+1}}\cdot \tag{30}$$

Using the fact that Γ is real, it is possible to express (30) in terms of S_0, the vswr measured with the second cavity detuned, and S, the vswr measured with both cavities tuned to resonance.

$$\frac{\gamma}{\gamma_c} = \sqrt{\frac{S_0 - S}{S}}. \tag{31}$$

Knowing the ratio γ/γ_c, and that the coupling coefficient varies as the cube of the hole diameter, it is possible to compute the hole diameter for critical coupling and hence for any value of coupling coefficient desired.

Besides obtaining the value of coupling coefficient experimentally, the conditions for a matched input looking to the filter can be derived. If (26) and (27) are solved simultaneously with the variables $(\omega M_1)^2$ and RZ_0, then the condition for the existence of a solution requires that

$$\Gamma_c = \frac{1 - 3\Gamma_0}{\Gamma_0 - 3}\cdot \tag{32}$$

Therefore, a good match with critical coupling requires that $\Gamma_0 = 1/3$. The more general case follows from setting $\Gamma = 0$ in (29) and solving for γ/γ_c.

$$\gamma/\gamma_c = \sqrt{S_0 - 1}. \tag{33}$$

This equation determines the coupling coefficient for $S = 1$ at the filter resonant frequency.